

Jari Pakkanen

# CLASSICAL GREEK ARCHITECTURAL DESIGN: A QUANTITATIVE APPROACH



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Jari Pakkanen

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## Preface

In many ways, this monograph is a methodological summary of the work I have carried out on Greek architectural design over the past twenty years or more. The case studies analysed here stretch from my first fieldwork project on the temple of Athena Alea at Tegea which started in 1993 to the on-going research collaboration on the city-scape of Naxos in Sicily. I took the first steps towards the statistical analysis of Greek foot-units in 1999 by developing a computer program to analyse building dimensions: some of the final motivation behind the software was to try to stop my post-graduate student Esko Tikkala from being drawn into the shapeless muddle of architectural studies on Greek foot-standards. I was soon nudged by Mike Baxter towards Kendall's cosine quantogram analysis as a useful statistical method and my work then evolved towards a quantitative analysis of Classical architectural design rather than just foot-units.

Three fieldwork projects are at the basis of case studies presented in Chapter IV. My work in 1993–1998 on Classical architecture at Tegea was part of the Norwegian excavation project directed by Erik Østby: I cannot thank him enough for the continuous support over the years. I directed and carried out the fieldwork at the temple of Zeus at Stratos under the auspices of the Finnish Institute at Athens in 2000–2001. The permissions to carry out the fieldwork at Tegea and Stratos were granted by the Hellenic Ministry of Culture and supported by the local Ephorates of Prehistoric and Classical Antiquities at Tripolis and Patras and their Ephors at the time, Theodoros Spyropoulos and Lazaros Kolonas. Since 2006 I have collaborated with Maria Costanza Lentini, the director of the Museum of Naxos in Sicily, on the architecture of the ancient town, and our new co-operation concentrates on the overall city design: I am very grateful to her for the chance to work at this unique site and the warm hospitality I have always been shown.

During the final phase of writing, the comments on the manuscript by Ann Brysbaert and Björn Forsén have been of especial value. The following persons have also read various stages of the chapters of this book, and their comments have been more than welcome: Jim Coulton, Manolis Korres, Seppo Mustonen, Erik Østby, Boris Rankov, Esko Tikkala and Richard Tomlinson. Various anonymous referees have had their input into this work, and I also wish to thank the publication committee of the Papers and Monographs of the Finnish Institute at Athens, Björn Forsén, Mika Kajava, Martti Leiwo and Manna Satama, for the prompt dealing with the book proposal and its publication.

The final impetus to write this book came from the British Academy and its mid-career fellowship: during the year I have enjoyed the possibility of concentrating on quantum analyses and not only in relation to Greek architecture. My home institution, Royal Holloway, University of London, has supported me in various ways over the years, including the current possibility of being seconded to the Finnish Institute at Athens where I have just started as the director. Financial support for the fieldwork case studies has come from the Academy of Finland, the Emil Aaltonen Foundation, the Finnish Institute at Athens and the Museum of Naxos. Numerous people have collaborated with me in one way or another in the fieldwork projects: Anne-Claire Chauveau, Øystein Ekroll, Anne Hooton, Christina M. Joslin, Marianne Knutsen, Jerrad Lancaster, Tara McClenahan, Petra Pakkanen, Thomas Pfauth, Tuula Pöyhä, Esko Tikkala and Rauno Vaara.

I have programmed the computer modules used in the cosine quantogram analysis, Monte Carlo simulations and producing the kernel density estimation distributions on top of the statistical package Survo MM. Cosine quantogram is now a standard feature of Survo, and the module implemented by Seppo Mustonen is far faster than the software I originally programmed.

The staff of the Finnish Institute has created a very positive working environment, and I am also very grateful to Maria Martzoukou, Saara Kauppinen, Maria Gourdouba and Joakim Stavropoulos for their help in all kinds of practical matters.

Without the continuous support of Ann Brysbaert this monograph would not exist. I dedicate this book to the memory of my father Ahti Pakkanen. In 2002, he located and sent me a copy of von Mises' article published in 1918: it is the first study which employs the sum of cosine values to determine the size of an unknown unit in a data set.

Jari Pakkanen  
At Athens, September 2013

## I. Introduction

It is perhaps to be expected that different scholars analyse similar measurement sets from the same building and come up with quite different hypotheses which type of a proportional or modular design scheme was employed by the ancient architect. Nevertheless, I find the current state of architectural design studies intriguing. Are we now in a situation where no consensus of how the fifth-century Greek architects worked out their designs can be reached? If this question is deemed out of our grasp, would it at least be feasible to evaluate the validity of the results obtained in previous design studies and give guidance for future research as to which approaches will most likely produce fruitful outcomes? I think the answer to the latter question is certainly positive, and with the publication of this book I hope there will be more clarity how an agreement among scholars can be achieved.

The ultimate aim of this book is to change the prevalent paradigm in Greek architectural design studies: detecting patterns in a set of measurements is to a high degree a statistical question and scholars who ignore this fact risk confusing the discussion rather than clarifying it. A number of scholars will perhaps find it difficult to accept some of the methodological critique and conclusions reached in this monograph. However, the foundations on which Greek architectural design analyses are built are not necessarily as stable as is often taken for granted: for example, studies which assume a high degree of foot-unit standardisation in the Hellenic world are in serious danger of reaching false conclusions. Recognising complex patterns in data sets requires expertise both in the field of the study in question and quantitative methods. It is possible to bring together specialists in architecture and statistics for a particular project,<sup>1</sup> but often real insights come from an interdisciplinary understanding of the relevant fields and over a long period of time. The critique I present here arises from twenty years' experience of systematic archaeological fieldwork on the Greek built environment and both subsequent and intertwined studies on how the emerging problems can possibly be solved by using computer-intensive statistical methods. The emphasis of this monograph is on methodology and my aim is not to give an exhaustive analysis and criticism of previous studies on Greek architectural design: given the number of suggestions for each monument this would be quite an impossible task. Taking relevant examples and examining them in detail is a far more productive approach.

In the introduction I present a brief overview of how Greek architectural design has been approached in previous scholarship and, thus, set the frame for the reasons behind the current lack of consensus. What emerges from the review is that before some general agreement can begin to form, it is necessary to find a more rigorous methodological approach to the questions at hand.

Chapter II looks first at what types of issues related to building design might be encountered in the statistical analyses. A quantitative method based on D.G. Kendall's cosine quantogram analysis (CQG) is also defined in the chapter, and in Chapter III this method is used to analyse a group of fifth-century Doric temples. There are significant benefits from initially analysing a relatively coherent group of buildings as a whole rather than concentrating on single ones.

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<sup>1</sup> See e.g. Coulton 1979 for a very significant early example.

Chapter IV presents four case studies ranging in size from the design of the Classical city grid at Naxos in Sicily to the moulding details of the temple of Athena Alea at Tegea.<sup>2</sup> The Erechtheion case study links the analysis of building dimensions with inscriptional evidence, and I use the temple of Zeus at Stratos as an example of how the plan and façade dimensions of a single building can produce varying levels of statistical significance. All these studies highlight different aspects how CQG can be used in the analysis of possible proportional and modular systems and potentially employed foot-units in Greek architectural design. One recurring theme is how the number of dimensions and the choice of data affect the results: different measurement sets can highlight different aspects of the original building design and execution, or sometimes also hide relevant patterns.

The conclusions of the monograph are presented in Chapter V. The first of the two glossaries at the end gives definitions for the architectural and the second for the statistical terms.

## Methodological Approaches to Greek Architectural Design

Previous studies on Greek architectural design have used slightly differing strategies in arguing how the Doric temples of the fifth century were planned. Frequently the analyses of single buildings start with the hypothesis that one or more of its dimensions, such as the intercolumniation, the width and the length of the building or the size of a typical wall block, can be expressed as round numbers or simple fractions of an ancient foot-unit. The approach can be called the ‘standard metrological method’ or ‘inductive metrology.’<sup>3</sup> Other dimensions are subsequently given in terms of the initially detected unit, but almost invariably small fractions are needed to guarantee a reasonably successful fit between the measurements and the foot-standard.<sup>4</sup> Frequently, the precision with which the length of the unit is expressed does not follow the rules on the number of significant digits. Most often, the relevant dimensions can only be established with the precision of three or four digits, such as the triglyph width or the column interaxial: the derived unit should not have more significant digits than the element with the least number of significant digits used in the calculations.<sup>5</sup>

Problems of relating actual building measurements to the two ‘standardized’ units, the ‘Attic–Ionic’ foot of c. 294 mm and the ‘Doric–Pheidonic’ foot of c. 326

<sup>2</sup> Three of the case studies have been previously published (Pakkanen 2004b; Pakkanen 2005; Pakkanen 2006–2007), but they have appeared in journals or books with limited circulation. Also, new material has been incorporated into the case studies on Stratos and Tegea. I have re-edited the original publications for this monograph and their analyses and conclusions have been updated.

<sup>3</sup> Pakkanen 2004a, 258; Dinsmoor and Dinsmoor 2004, 5.

<sup>4</sup> The Dinsmoorian analysis of the Propylaea is a good example: the length of the ‘Doric’ foot-unit is derived from the wall blocks as 0.32723 m, and the smallest fraction used in their metrological analyses is 1/192 of this unit (0.0017 m); Dinsmoor and Dinsmoor 2004, 5–7, 447–449. Eiteljorg (2005, 41–44) has recently argued that on the basis of the wall blocks the unit should be defined as 0.295 m, so that it is the ‘Attic’ foot which is used in the building rather than the ‘Doric.’

<sup>5</sup> E.g. in case of triglyph width of, say, 0.647 m and an interaxial of 1.632 m, it can be suggested that the related foot-unit is  $0.647 / 2 \approx 0.324$  m or  $1.632 / 5 = 0.3264$  m. It is incorrect to propose that the length of the unit could be calculated as the average result,  $(0.3235 \text{ m} + 0.3264 \text{ m}) / 2 \approx 0.3250$  m. Based on the two dimensions it can only be proposed that they point towards a possible foot-standard in the range 0.324–0.326 m.

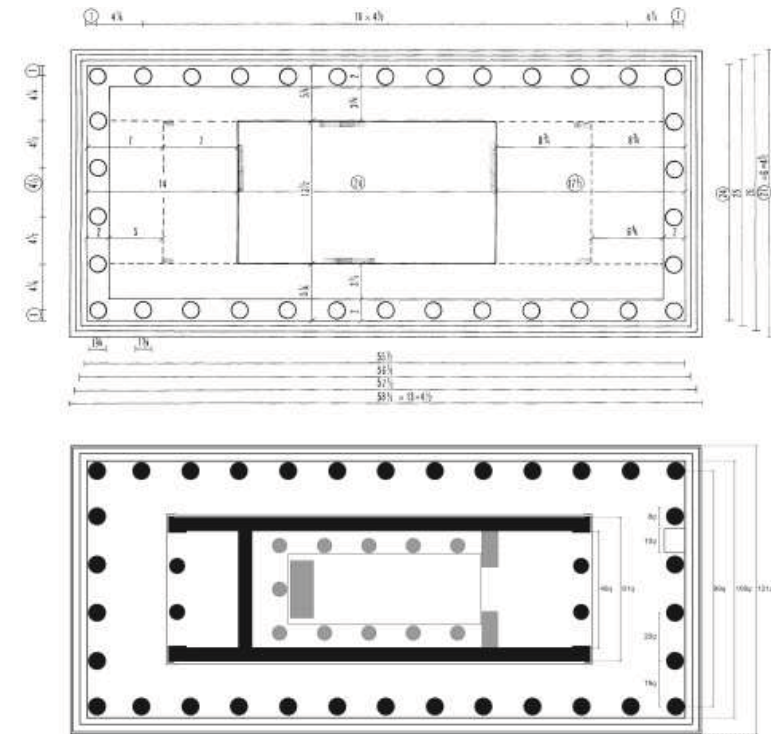


Fig. 1.1. Hypothetical regular and realised plans of the Hephaisteion (top: based on Zwarte 1996, fig. 2; below: J.P., based on the quantum detected in the façade analysis of Chapter III and Camp 1986, fig. 59).

mm,<sup>6</sup> have resulted in a number of modified approaches and reactions against the standard design analyses. One way of reconciling the differences has been to assume a hypothetical regular scheme for the building and explain the observed discrepancies as the results of modifications to the original regular plan (Figure 1.1).<sup>7</sup> H. Bankel’s ‘metrological scale’ makes it possible to study simultaneously how well a number of architectural dimensions fit to particular subdivisions of feet:<sup>8</sup> the benefit of the method is that it does not start with a presupposition of the size of the foot-unit, but as a graphic method only a very limited number of measurements can be studied in one drawing.<sup>9</sup> Several

<sup>6</sup> The most frequently quoted analysis on recognizing two principal units in Greek architecture is Dinsmoor 1961.

<sup>7</sup> See e.g. Riemann 1951; Riemann 1960; Zwarte 1996. Wilson Jones (2001, 677) rightly observes that there is no need to link changes of the type hypothesized by Riemann to a single project but that they rather characterize a development over a longer period of time.

<sup>8</sup> See e.g. Bankel 1983; Bankel 1984. For an example of his metrological scale, see Figure 4.20 below.

<sup>9</sup> Both Bankel’s methodology and his results have been criticised; see Wesenberg 1984, 549–553; Büsing 1985, 159–160; Pakkanen 2004b, 111–119; Pakkanen 2005, 170–173. See also pp. 68–70 and 95–97.

scholars have recognized the fact that the question of the degree of standardization of measurement units in the Greek world has not been satisfactorily solved<sup>10</sup> and suggested alternative approaches. One has been to continue using the standard metrological method but accept non-standard units as possible results of the analyses. The most radical hypothesis has been proposed by W. Koenigs: his suggestion is that every single monumental Greek building employs a measurement-unit particular to that design scheme. The length of this 'Iochmodul' is related to the interaxial column spacing.<sup>11</sup> Since Koenig's core idea is that the derived length is a non-standard measurement-unit, J.J. Coulton argues that an architecturally more correct term for the Iochmodul would be 'Iochfuss'.<sup>12</sup>

This standard metrological approach to Greek design has resulted in quite a bewildering picture what the length of measurement or design units employed in each particular building is. The temple of 'Hera Lakinia' at Akragas can be listed as a typical rather than an extreme example:

1. A foot-unit of 296 mm is suggested by H. Riemann based on principal plan dimensions expressed in round numbers of feet, though also smaller subdivisions are needed.<sup>13</sup>
2. A foot-unit of c. 307 mm is mostly used by J.A.K.E. de Waele in his analysis, and I. Ceretto Castigliano and C. Savio express the principal plan dimensions in round numbers of feet of this unit, but they need to also resort to smaller subdivisions. Ch. Höcker gives his unit as 307.2 mm.<sup>14</sup>
3. Rather confusingly, de Waele also suggests that the cella design fits better a unit of c. 320 mm than his other unit.<sup>15</sup>
4. A 'Doric' foot of 325.86 mm is derived by D. Mertens from the stylobate width, but then smaller subdivisions of this foot are needed to match it to the other analysed elements.<sup>16</sup>
5. An oversize 'Doric' foot of 328.8 mm is suggested by M. Wilson Jones based on his 'triglyph module' of 616.5 mm;<sup>17</sup>

Since a similar situation is encountered in relation to nearly all monumental Greek buildings of the fifth century BC, it is no wonder that it has not been possible to build up a consensus regarding the architectural design principles of the period.

Wilson Jones has relatively recently published a highly developed argument that the Doric façade design was based on a module derived from the triglyph width. This hypothesis is studied in detail in Chapter III, so it is useful to present his criteria for establishing the design scheme of a Doric temple. It is in essence a variant of the standard design analysis method:

1. Start with the actual triglyph width.

2. Find an approximate unit which fits one fifth of the normal column interaxial and produces a neat pattern when compared with the principal plan and elevation dimensions; this is the potential design module.
3. Control that the module relates to the schematic proportions of the whole façade and that the proportions expressed in terms of the module are convenient.
4. Control that the module can be given in terms of known feet and/or dactyls.
5. Check the relationship between the smaller components of the façade and the module.
6. Check similarly the plan.<sup>18</sup>

The proposed procedure sounds perfectly reasonable, but it is methodologically highly problematic. For example, taking into consideration how close to each other the notions of the Iochfuss (or Iochmodule) and Wilson Jones's ideal triglyph width are, both of them can surely pass the tests put forward by Wilson Jones.<sup>19</sup> The criteria cannot compensate for the basic problem inherent in standard design analyses. The method simply cannot adequately differentiate between the possible design strategies the fifth-century architect could have employed: using quite similar sets of architectural measurements it is possible to argue that Doric temple designs are based on a modular design system starting with the triglyph width, foot-standards or modules derived from the intercolumniation, standard or non-standard foot-units, or on a non-modular design system based on proportional relationships between the elements, or a successive design system. Even if a particular study is on the right track, the scholar has no tools with which to convince the others that this is the case. Hypotheses of several scholars are supported by the statistical analyses presented in this monograph, but they are perhaps not among the most widely cited ones in Greek architectural studies. Koenigs' and Coulton's Iochfuss-hypothesis can be given as an example, and Höcker has observed that in four of the six classical temples at Akragas a module based on one tenth of the intercolumniation produces systematically relatively good results when the principal dimensions of the buildings are expressed in terms of this module; for the two remaining buildings he identifies the module as one eighth and one twelfth of the interaxial distance.<sup>20</sup>

Even though the traditional analyses of architectural design and ancient metrology do not use mathematical methods to any great extent, all these studies employ a system where the discrepancies between the advocated design system and the actual measurements taken of the buildings are calculated. Furthermore, these calculations are then used to argue for the validity of the proposed system – surely the earlier a proper quantitative method is introduced into the analysis, the less scope there is for an incongruous interpretation being put forward. For example, I think Wilson Jones's basic aim in his Doric design studies can fairly be summarized as trying to find a general trend in ten different sets of architectural measurements.<sup>21</sup> The starting point of the research is a large and complex set of dimensions, and based on previous scholarship we can assume that a number of different types of factors could have had a role in shaping them.<sup>22</sup> It is

<sup>10</sup> The sceptical attitude is best summarized by Coulton (1974, 62): 'As far as measurement is concerned, the assumption that only two foot-standards were used throughout the Greek world needs to be proved, not just accepted, and the chaotic situation in other branches of Greek metrology suggests that this is unfounded.'

<sup>11</sup> Koenigs 1979.

<sup>12</sup> Coulton 1989, 87.

<sup>13</sup> Riemann 1935, 149.

<sup>14</sup> Waele 1980, 219–222; Ceretto Castigliano and Savio 1983, 36; Höcker 1993, 89–93.

<sup>15</sup> Waele 1980, 220–222.

<sup>16</sup> Mertens 1984, 105–107. Dinsmoor (1961, 360) very probably supports this identification: the temple is not specifically mentioned by him, but it is likely in the group of 16 non-named examples – according to him the unit of 326–7 mm was in use throughout the Sicilian Greek colonies.

<sup>17</sup> Wilson Jones 2001, 707.

<sup>18</sup> Wilson Jones 2001, 683.

<sup>19</sup> This has been also observed by G. Waddell who uses a variation of the standard method to argue that the design unit was derived from the krepis dimensions (Waddell 2002).

<sup>20</sup> Module identified as  $1/10$ : the temples of 'Concord', 'Hera Lakinia', the 'Discuri' and Temple L; as  $1/8$ : Temple E; as  $1/12$ : temple of Hephaistos; see Höcker 1993, 79–117.

<sup>21</sup> Wilson Jones 2001; Wilson Jones 2006.

<sup>22</sup> For the most in depth discussion of these factors, see Coulton 1975a.

generally agreed that the sizes of the various building elements are proportionally related to each other and scholars tend to regard it as very likely that a number of dimensions were rounded to the nearest comfortable fractions of the employed measurement-unit. This often creates, however, difficulties to determine what the exact relationship between the original design and the finished structure is. Also, substantial variation in the size of identical elements was tolerated by the ancient architects and builders, as can perhaps be expected due to the nature of the building process and materials used. In addition, the buildings' state of preservation is often not perfect, and errors in modern measurements and their publication cannot be excluded. It is unlikely that any scholar is able to take into account simultaneously all these factors by testing various alternatives with a pocket calculator, so in trying to make sense out of complex data sets and evaluating the relevance of different types of factors a solid mathematical method is of vital importance.

A more extensive overview of the various design features identified in earlier research is given on pages 9–14, but there are two topics which are not usually given the emphasis they would deserve in architectural design analyses. The question of what types of foot-standards were employed in Greek building has not been satisfactorily solved. The disagreements among the scholars could be due to lack of general standardization,<sup>23</sup> but the situation has not been helped by the use of inappropriate methods to tackle the issue.<sup>24</sup> Deriving the length of Greek measurement-units from a set of building dimensions is a problem which can best be approached by statistical means, as is demonstrated in this monograph. Wilson Jones shows that similar proportional relationships between different parts of a building could hypothetically be a result of either employing a proportional design scheme for the façade or equally well a system based on a strictly defined module,<sup>25</sup> but I think that Coulton's suggestion that Greek architects probably used a successive system of proportion should be given more consideration than is generally done in design studies (Figure 1.2). Coulton demonstrates that there is a clear difference between Vitruvius' definitions for the Doric and Ionic orders. The Doric has a radial pattern with most of the dimensions derived fairly directly from the module, while the Ionic is much more linear.<sup>26</sup> Following Vitruvius' rules for the Doric produces a transparent design, and establishing the relationships between the various parts of the building should be quite a simple task involving not much more than testing the different possibilities with a calculator or a spreadsheet. Analysing a successive system such as Vitruvius' Ionic can be a much more difficult exercise. If the dimensions are rounded at each step, it is possible that the sizes of the elements higher up in the façade bear no precise proportional relationship to the size of the module: the whole design is likely to be far more opaque than the Doric. A successive system is also compatible with the evidence for incomplete preliminary planning of fifth-century Greek monumental architecture which is best exemplified by the frequent problems the architects had in designing the sizes of the frieze elements.<sup>27</sup>

<sup>23</sup> Cf. Coulton 1974, 62; Waele 1988, 205–206; 1990, 1; Cooper 1996, 131–132; Wilson Jones 2000, 75.

<sup>24</sup> See Pakkanen 2004a, 258; Pakkanen 2005, 167–174.

<sup>25</sup> Wilson Jones 2001, 680–682.

<sup>26</sup> Vitr.4.3.3–10, 3.5.1–15; Coulton 1975a, 68–73; Coulton 1977, 66–67. Successive systems also fit Coulton's (1975a, 64) idea of governing and defining factors in Greek architectural design: 'We can, however, suppose that the dimensions of each part of a building were governed by a system of proportion, but defined by being rounded out to the nearest convenient dimension. That might explain, theoretically at least, the general uniformity of Doric proportions and at the same time the infrequency of simple ratios.'

<sup>27</sup> Coulton 1975b; Coulton 1977, 60–64.

Statistical studies can clarify the issue. As will be shown in Chapters III and IV, CQG analysis provides a robust method for detecting signals resulting from both proportional relationships and rounding of building dimensions to the nearest convenient fraction of a foot. Analyses suggest that a new definition of 'modular' can be put forward: there are good grounds to call an architectural design being based on a single module in cases which are transparent enough to produce a single statistically significant peak in the CGQ analyses. For the other buildings an alternative explanation needs to be sought, so perhaps their scheme was non-modular or their execution did not precisely follow the initial design.

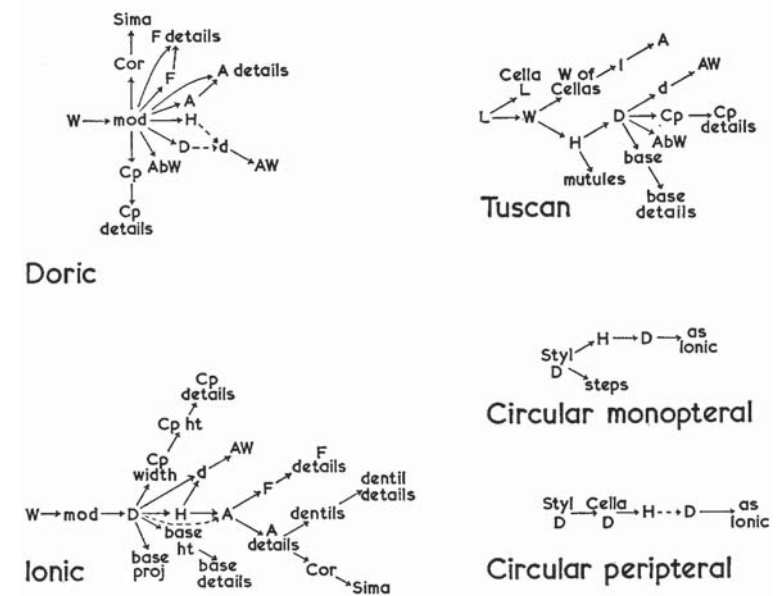


Fig. 1.2. Modular and successive design systems in Vitruvius according to Coulton 1975a, fig. 1.



## II. Method

The method used in this monograph is very different from the standard approach to architectural design principles and metrology: establishing how the Greek architects worked out their designs is a far more complex task than is taken for granted in most of the earlier scholarship.<sup>1</sup> The Parthenon is a good example of the confusion created by the standard method. Different scholars have suggested a whole array of foot-standards, modules or cubits which can be fitted to the building dimensions: 29.366–29.436 cm, 30.5–30.7 cm, 32.6–32.8 cm, 49.02857 cm, and 61.2857 cm.<sup>2</sup> Most of the studies are accompanied by a proportional design scheme which can be fitted to modern measurements more or less successfully. If the standard method of design analysis was not used as widely as it is in architectural studies, it would be needless to point out that it is not a valid method of conducting research. I argue that if our aim is to understand how architects designed their projects in ancient Greece in the Classical period, a proper statistical analysis should be an essential part of all metrological and design studies.

### What to Look for in the Design Analyses?

Before defining the quantitative approach used in this book it is necessary to discuss what types of possible design patterns it is indispensable to keep an eye on in the analysis of the statistical results.

The most important single ancient source on Greek classical foot-standards is Herodotos: from the fifth-century historian we learn that different foot lengths were in use, and he discusses some of the relationships between the different units.<sup>3</sup> Unlike the later Roman foot which was divided into twelve inches, the Greek foot was divided into four palms and a palm into four dactyls or finger-breadths. Contrary to the well documented Roman foot,<sup>4</sup> the lengths of suggested Greek units are usually derived from analyses of building dimensions. Some indications on the lengths of the standards may possibly be derived from two preserved metrological reliefs<sup>5</sup> and combining the information of a length given in an ancient inscription with the actual measurement of the element.<sup>6</sup> There is some evidence, both empirical and textual, that ancient building dimensions could have been rounded to the nearest full feet or simple fractions of feet.<sup>7</sup> As we saw in the example given in the introduction on the temple of 'Hera Lakinia' at Akragas, the possibility of expressing the principal plan dimensions in round numbers of feet is a key point in the traditional arguments on the length of foot-units employed by the architect. However, this cannot be taken for granted, and the observation that the dimensions can conveniently be expressed in round numbers should be an end result of the analysis, not part of its premises. The Greek architectural inscriptions rarely refer to

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<sup>1</sup> For further criticism and evaluations of previous metrological studies on Greek architecture, see Pakkanen 2002; Pakkanen 2004a; Chapters III and IV in this book.

<sup>2</sup> Sonntagbauer 1998, 136.

<sup>3</sup> Hdt. 1.60, 1.178, 2.149, 2.168, 6.127.

<sup>4</sup> See e.g. Röttlander 1993, 85–107.

<sup>5</sup> Michaelis 1883, 335–350; Dekoulakou-Sideris 1990, 445–451; Slapšak 1993, 119–136; Wilson Jones 2000, 73–93.

<sup>6</sup> See e.g. Haselberger 1983, 115–121; pp. 14–20 and 60–64 below.

<sup>7</sup> Coulton 1975a, 62–65, 85–89.

fractions smaller than a quarter foot.<sup>8</sup> For example, the building blocks of the still incomplete Erechtheion at Athens were inventoried in the late fifth century, and most dimensions are expressed in terms of either full or half feet (see also Table 2.1 below).

Simple arithmetical proportions are one effective way of designing a well-balanced building and their use could also explain the slight changes from one building to the next.<sup>9</sup> The use of simple fractions is very conspicuous in Vitruvius, and from his references we know that the practice went back at least to the Greek second-century BC architect Hermogenes and his Ionic buildings, but very likely Hermogenes was using much earlier design principles.<sup>10</sup>

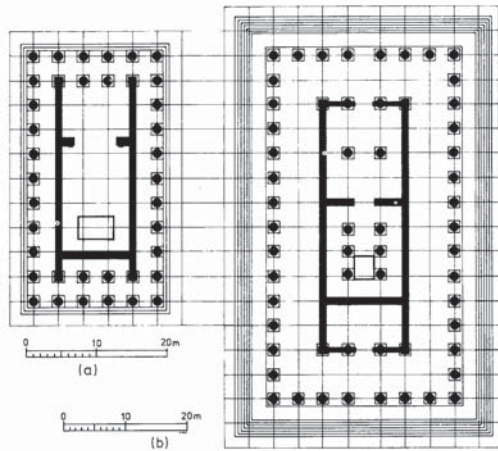


Fig. 2.1. Ionic temple plans imposed on a uniform grid. On the left the temple of Athena at Priene and on the right the temple of Artemis at Magnesia (Coulton 1977, fig. 23).

Architectural scholars agree that a modular design pattern can quite easily be detected in fourth-century and Hellenistic Ionic architecture: the temples of Athena at Priene and of Artemis at Magnesia are typical examples (Figure 2.1).<sup>11</sup> Both temple plans are strictly modular, and the axes of the surrounding colonnade and the cella walls match very closely. In the temple of Artemis the central bays at the front and the back are wider than others, but the cella design follows this wider spacing. Coulton's proposition that successive proportional design schemes could be an important factor behind the opacity of Doric architectural design was discussed in the introduction (page 6 and Figure 1.2): successive systems can be far less transparent than modular ones.

<sup>8</sup> Coulton 1975a, 92–93.

<sup>9</sup> See e.g. Coulton 1977, 64–68; Mertens 1984, 104–105.

<sup>10</sup> Vitr. 3 and 4; on Hermogenes, see Vitr. 3.3.8. Cf. Coulton 1975a, 63; Coulton 1977, 70–71. In general on the architect, see Hoepfner and Schwandner 1990.

<sup>11</sup> See e.g. Coulton 1977, 71; Wilson Jones 2001, 675–676.

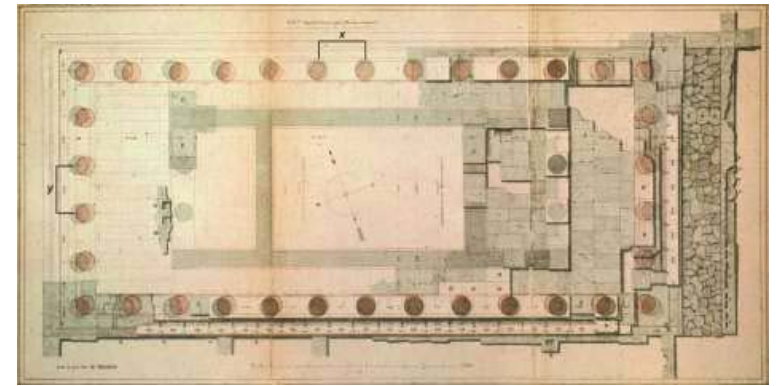


Fig. 2.2. Superimposed plans of the Archaic and Classical temples of Poseidon at Sounion (Dörpfeld 1884, pl. 15). The equal interaxials on the long and short sides of the Archaic temple marked with *x* and *y*.

In most fifth-century-BC Doric architecture the stylobate proportions seem to be secondary to column spacing.<sup>12</sup> Remains of the Old Temple of Poseidon are incorporated as part of the foundations of the later temple at the site (Figure 2.2). In the Old Temple of Poseidon the front and flank interaxials are the same, so the architect obviously fixed this distance first and sacrificed the overall proportions at the stylobate level.<sup>13</sup> Koenigs has suggested that monumental Greek buildings were designed using a measurement unit unique to each building.<sup>14</sup> This is not as an unlikely proposition as it initially sounds and could explain why there is so little consensus what comes to fifth-century Doric design. The length of this 'Iochmodul' or 'Iochfuss' would have been derived from the interaxial column spacing. One practical result of this suggestion is that it would be nonsensical to try to derive the standard foot-unit lengths from building dimensions: this has been one of the common approaches in Greek architectural analyses, so Koenigs' hypothesis needs to be taken very seriously.

Taking building dimensions and expressing them in terms of possible Greek measurement units does not advance our understanding of Greek architectural design, and the reason is simple: almost any metric dimension can be expressed quite well in terms of at least one of the proposed units. The unit selection and measurement ranges presented here are based on a relatively recent article, and it shows the situation around the one meter mark (Figure 2.3).<sup>15</sup> The darker vertical zone indicates the only measurement range which cannot be given as feet and dactyls as defined in this particular study, and the lighter area gives the short stretch not covered by the 'Attic' and 'Doric' units. So, for example, a dimension measured as 0.98 m could be three 'Attic' feet and five dactyls, three 'Doric' feet, two 'Samian' feet and thirteen dactyls, or three 'common'

<sup>12</sup> Coulton 1974, 74–77.

<sup>13</sup> Coulton 1974, 74; Dinsmoor (1950, 107) notes that the stylobate had 'simple dimensions of 40 by 92½ feet', but the dimensions do not translate into a simple proportion as was typical for sixth-century temples.

<sup>14</sup> Koenigs 1979. See also Coulton 1974, 62; Waele 1988, 205–206; Waele 1990, 1; Cooper 1996, 131–132; Wilson Jones 2000, 75.

<sup>15</sup> Wilson Jones 2001. For a more thorough discussion, see pp. 67–68.

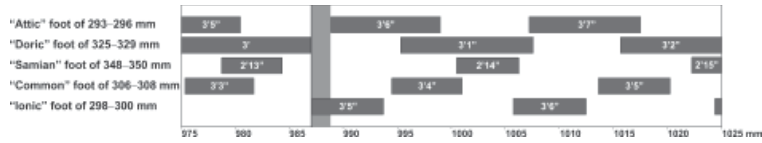


Fig. 2.3. Standard Greek foot-units and measurements in the range 0.975–1.025 m.

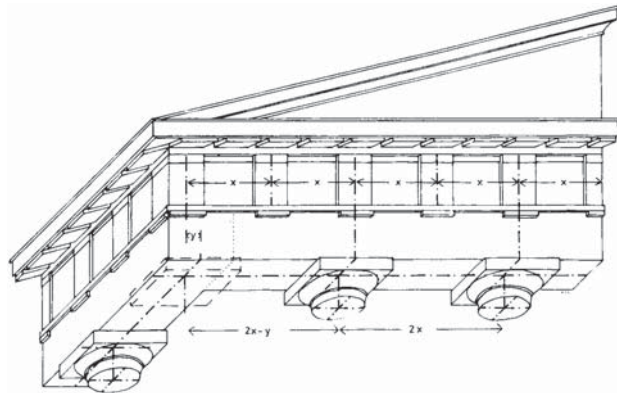


Fig. 2.4. Hypothetical design of a regular Doric frieze (Coulton 1977, fig. 18).

feet and three dactyls. Stretching the size and number of possibly used foot-units renders most metrological analyses rather empty exercises. But again, I hope to show in this monograph that an approach which does not start with predefined notions of the foot-unit sizes can actually avoid the dangers underlined here.

The layout of a Doric building is rarely as transparent as an Ionic one. This is due to the relationship between the columns and the frieze. In a hypothetical wooden peristyle there would have been no corner conflict since the triglyph width matches the architrave depth.<sup>16</sup> In stone buildings the depth of the architrave is greater than the triglyph width, so either the frieze elements have to be stretched or the distance between the two corner columns shortened to produce a regular frieze (Figure 2.4). What can be observed in Classical Greek buildings is a compromise: the architects most probably used a thumb rule for placing the columns which, in most cases, produced a more or less regular frieze, but almost always some adjustment in the width of the frieze elements was required.<sup>17</sup> However, at least by the end of the fourth century there were architects who were willing to test whether Ionic modularity could be introduced into Doric buildings: the temple of Zeus at Stratos in western Greece is analysed in detail on pages 75–93 below.<sup>18</sup>

<sup>16</sup> See e.g. Gruben 2001, fig. 25.

<sup>17</sup> Coulton 1974, 72–73; Coulton 1975b, 15–16; Coulton 1977, 60–64.

<sup>18</sup> For the most recent overview of the general trends in Hellenistic temple design, see Sioumpara 2011, 244–263.

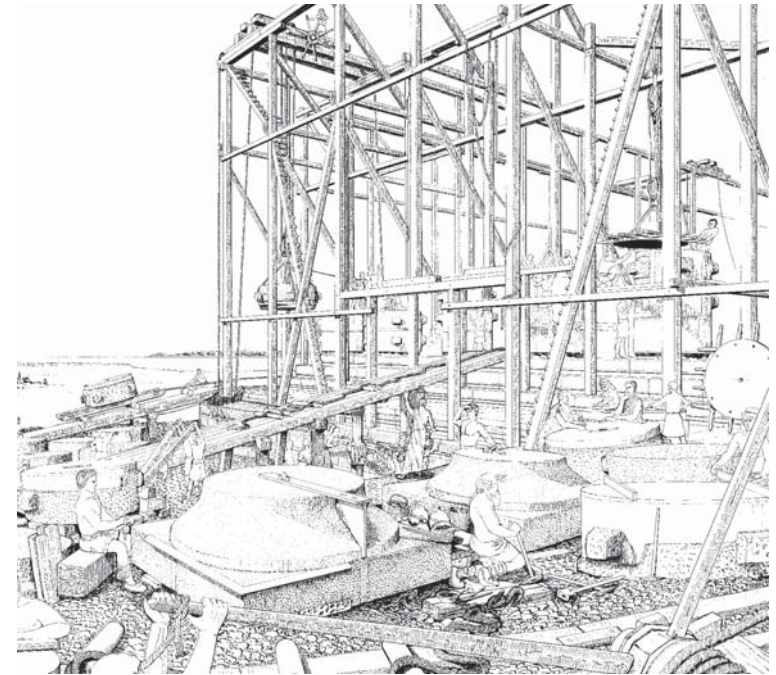


Fig. 2.5. Hypothetical reconstruction of the construction site of the first marble Parthenon (Korres 1995, pl. 19).

One very significant factor in the analysis of an original building design is the degree of precision which can be expected in the finished building. The architects and construction workers tolerated quite substantial variations in the execution of their monumental buildings. This is not surprising since all the blocks were finally cut to size at the building site and fitted to the stones next to it (Figure 2.5). Variation of  $\pm 0.01$  m between similar smaller architectural elements is quite typical of Greek building practice.<sup>19</sup> However, even the Parthenon on the Athenian Acropolis provides several larger examples, such as the abacus width of the normal column capitals which varies by almost 6 cm,<sup>20</sup> and the variation in the length of the five architrave blocks on top of the normal column bays of the east front of the Parthenon: they should all be of equal length, but the difference between the shortest and longest block is 0.18 m. The bays vary only by 0.01 m, thus causing the architrave joints to be significantly off the alignment of the columns.<sup>21</sup> J.A. Bundgaard suggests that the differences in block lengths are explained by the reluctance of the masons to cut away more than was

<sup>19</sup> Coulton 1975a, 94.

<sup>20</sup> 1.997–2.055 m; Balanos 1938, 38.

<sup>21</sup> Balanos 1938, dépliant no. 10.

absolutely necessary of the blocks coming from the quarry: the four largest blocks were probably used to the full and only the shortest block cut down.<sup>22</sup> Quite often these examples have been overlooked even in modern studies, and the precision of the workmanship – for example, the jointing of blocks is very accurate – is taken to apply to all of the building.<sup>23</sup>

There is wide agreement between scholars of Greek architecture that proportional systems relating the size of the building elements to each other were used by Greek architects and that these proportional relationships can be detected by studying the actual dimensions of the building. Proportional analyses often seem rather convincing, but one question invariably rises: how precisely should the actual building dimensions fit to the presented scheme? This basic question is rarely satisfactorily solved in architectural studies. It naturally varies from building to building depending on the ancient execution, current state of preservation and the skill and tools of the modern scholar. This question will be one of the recurring themes in this book.

## Statistical Methods

The approach adapted in this book was originally published in 1974 by D.G. Kendall. Basically, it is a statistical analysis of whether the hypothesis that the diameter of British megalithic circles was designed using a measurement unit of 1.66 m is supported by the measurement data;<sup>24</sup> it should be emphasised that Kendall's study is a methodological one and that the modern understanding of the archaeological significance of the original question is distinct from the proposed method. His approach is built on earlier studies by R. von Mises and S.R. Broadbent how to detect a quantum of unknown size in a set of measurements, but his cosine quantogram (CQG) method proposes a modified algorithm for the data analysis; he also suggests that the validity of the obtained results needs to be assessed by Monte Carlo computer simulations.<sup>25</sup> Revisions to Kendall's method have recently been suggested by the author of this monograph and by E. Çankaya and N.R.J. Fieller.<sup>26</sup>

Going through a relatively simple and concrete metrological example is the best way to understand how CQG can be used to study architectural design principles, and I will use the Erechtheion block measurements as such an example.<sup>27</sup> Several currently existing marble blocks can be identified in the late fifth-century BC building inventory

<sup>22</sup> Bundgaard 1957, 140–141.

<sup>23</sup> On variation and precision in Greek building more in general, see Coulton 1975a, 89–98.

<sup>24</sup> Kendall did detect a 'real quantum' in the data, but he demonstrated that it could equally well be a result of laying out the relevant dimensions by pacing (Kendall 1974, 258); a synopsis of the discussion is presented by Renfrew and Bahn (2000, 401) and Baxter (2003, 235) sums up the argument as follows: 'Although the megalithic yard may be dead, the methodology that some regard as having buried it lives on.' Several more relevant examples of quantal problems in archaeology are discussed in Fieller 1993, 282–286. Kendall 1977 is mainly a reprint of the original article, but on pp. 156–159 he presents a reply to some of the comments on his method.

<sup>25</sup> Mises 1918, 490–500; Broadbent 1955, 45–57; Broadbent 1956, 32–44; Kendall 1974, 234. A review of Kendall's method with a larger number of executed simulations is presented in Baxter 2003, 228–235. On the use of Monte Carlo methods in archaeology, see Pakkanen 1998a, 54–55, esp. n. 18; Baxter 2003, 147–158.

<sup>26</sup> Use of kernel density estimates to produce non-quantal simulation distributions: Pakkanen 2002; Pakkanen 2004a; multimodality: Çankaya and Fieller 2009.

<sup>27</sup> For a previous analysis of the data, see Pakkanen 2006–2007.

Table 2.1. The Erechtheion building block dimensions in *IG I<sup>3</sup> 474* (for sources of the dimensions in column 3, see Pakkanen 2006, table 1).

1. Block		2. <i>IG I<sup>3</sup> 474</i> (feet)	3. Measured dimension (m)
4 wall blocks	L (lines 10–11)	4	1.30
	W (line 11)	2	0.652
	H (lines 11–12)	1½	0.490
5 epikranitis blocks	L (lines 16–17)	4	1.301
	H (lines 17–18)	1½	0.492
Corner epikranitis	W (line 20)	4	1.242
	H (lines 20–21)	1½	0.492
8 architrave blocks	L (lines 33–4, 37–8)	8	2.608
	W (lines 34–5, 38–9)	2¼	0.77
	H (lines 35, 39)	2	0.63
3 Karyatid Porch roof blocks	L (lines 87–8)	13	4.200
	W (lines 88–9)	5	1.648
East frieze block	L (lines 115–19)	6 & 8	1.940 & 2.675
	W (lines 115–27)	1	0.285, 0.315
	H	2	0.617
North frieze block	W (lines 115–27)	1	0.298
	H	2	0.683
Geison block	L (lines 128–55)	4	1.301
	W	3	0.998

*IG I<sup>3</sup> 474*: their dimensions are given in the inscription and these can be compared to modern measurements. The full results of the Erechtheion analysis are presented in detail on pages 60–74. I start by taking the block dimensions listed in column 3 of Table 2.1 and subject them to independent statistical analysis: this means that the information given in *IG I<sup>3</sup> 474* is solely used to select the analysed blocks and the data on their size in feet are disregarded at this stage. This gives a set of measurements which should have an underlying basic dimension, a foot-standard in this case, which produces the observable lengths. In statistical terms this dimension is called a quantum; in the case of the Erechtheion the 'quantum hypothesis' is that a block dimension  $X$  can be expressed as the product of an integral multiple  $M$  times the quantum  $q$  plus an error component  $\varepsilon$ . In mathematical terms this can be denoted as

$$X = Mq + \varepsilon. \quad (1)$$

The critical factor in the formula is error  $\varepsilon$ : it sets a lower limit for quantum  $q$ . In any case  $\varepsilon$  or  $q - \varepsilon$  should be substantially smaller than any considered  $q$ .<sup>28</sup> Variation of  $\pm 0.01$  m between similar smaller architectural elements is quite typical of Greek building practice,<sup>29</sup> but by computer simulations it can be demonstrated that an error of this size has no effect on detecting a quantum in the region of c. 0.08 m, or approximately one quarter of a 'Doric' foot, even when the number of analysed building dimensions is small.<sup>30</sup> The case study on the Classical grid layout at Naxos in Sicily demonstrates that CQG can find a statistically highly significant design module of c. 1.6 m in a group of 40 measurements even with an average discrepancy of c. 0.2 m in the

<sup>28</sup> Since  $\varepsilon$  has a value between 0 and  $q$ ,  $q - \varepsilon$  can also produce an error significantly smaller than  $q$ .

<sup>29</sup> Coulton 1975a, 94. From a statistical point of view it does not matter whether the observed variation is due to factors in Greek building design and execution, the current condition of the blocks or modern measurement errors.

<sup>30</sup> Cf. Pakkanen 2002, 502–503; see also p. 20 below.

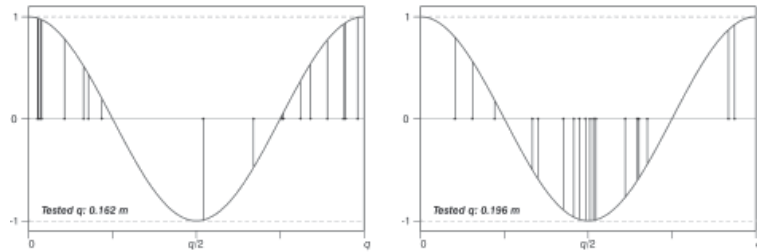


Fig. 2.6. Two examples of cosine values produced by dimensions in Table 2.1 Column 3.

executed dimensions. In the case of the Erechtheion, in order to give due consideration to units slightly smaller than a ‘normal’ quarter-foot or palm, I use a range of 0.06–0.40 m in the following analyses. The upper end is chosen so that it is greater than any suggested Greek foot-standard.<sup>31</sup>

In order to determine how well a block measurement  $X$  can be expressed in terms of quantum  $q$ ,  $X$  needs to be divided by  $q$  and the remainder  $\varepsilon$  analysed. The value of  $\varepsilon$  will be between 0 and  $q$ , and the less it deviates from either 0 or  $q$ , the better the fit between  $X$  and  $q$ . In Kendall’s cosine quantogram analysis  $\varepsilon$  is first divided by  $q$  and then the cosine of the quotient  $\varepsilon/q$  is taken: this gives a value of +1 for dimensions  $X$  which are an exact multiple of  $q$ , and the worst fitting measurements produce a value of  $-1$ . Figure 2.6 presents on the left the cosine values by the well-fitting  $q$  value of 0.162 m and on the right the pattern produced by the ill-fitting value of 0.196 m.

To find out which  $q$  is the best candidate for the quantum, it is necessary to compute the cosine value for all the measurements  $X$  and the full quantum range. How well the tested  $q$ -values fit the data can be determined from the cosine quantogram where the sum of the cosine values is plotted against  $q$ : the highest observable peak in the graph is the most likely quantum candidate (Figure 2.7). All this can be expressed as the following mathematical function  $\phi(q)$  for calculating the quantum score:

$$\phi(q) = \sqrt{2/N} \sum_{i=1}^N \cos(2\pi\varepsilon_i/q), \quad (2)$$

Here  $N$  is the number of measurements and the first term  $\sqrt{2/N}$  a scaling factor: in order to avoid getting a higher value for  $\phi(q)$  by simply introducing more measurements, the cosine sum needs to be scaled down.<sup>32</sup>

Figure 2.7 presents the CQG plot of the measurement data in Column 3 of Table 2.1. There are two apparent peaks, the first at 0.162 m and the second almost exactly twice the first at 0.325 m; the first corresponds obviously to the half-foot of the employed unit and second to the full foot. Since the statistical analysis makes no a

<sup>31</sup> The ‘Samian’ foot of c. 0.35 m; for references to the foot, see Wilson Jones 2000, 75 n. 16. There is no need to consider longer units such as cubits in this case since based on *IG I<sup>3</sup> 474* it is known that a foot-standard rather than anything substantially longer was employed in the construction process.

<sup>32</sup> Kendall 1974, 235–239.

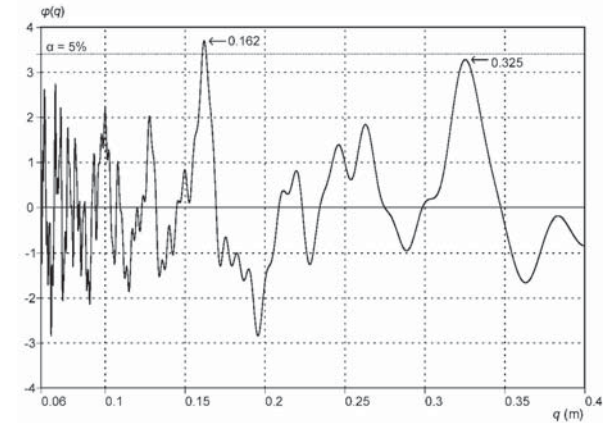


Fig. 2.7. CQG plot of the Erechtheion building block measurements listed in Table 2.1. Column 3.

*priori* assumption about the quantum size, or even its existence, it is highly significant that the cosine quantogram method points towards a slightly shorter unit than the current consensus on the length of the ‘Doric’ foot. The quantum score of the first peak is 3.70 and the second significantly less, 3.28. The next task is to find out whether the peaks are sufficiently high to be considered real quanta and not just background noise; if they are, then it would be convenient to know how precisely the length of the unit can be defined on the basis of the building block measurements.

The best means of evaluating whether the highest quantum score produced in the initial analysis is statistically significant is to build mathematical models of the data and use them to produce random non-quantal simulation data sets. These should have the same general statistical properties as the original set of measurements but lack the quantal ones. The replica data sets are then analysed in the same way as the primary data, and if the simulated function peaks are systematically lower than in the initial analysis, it is possible to accept the quantum hypothesis: the highest original peak can in that case be regarded as a valid candidate for the quantum<sup>33</sup> and directly related to the foot-standard used in the Erechtheion. Due to the random nature of the computer simulations, the method for testing the validity of the results is often called Monte Carlo analysis.

I have previously proposed that kernel density estimation (KDE) distributions are an effective way of producing the non-quantal data sets needed in the simulations.<sup>34</sup>

<sup>33</sup> Kendall 1974, 241–249; Fieller 1993, 282–283; Pakkanen 2002, 501–502; Baxter 2003, 231–233; Pakkanen 2004a, 263–270.

<sup>34</sup> On KDE distributions used as mathematical models to produce non-quantal data sets, see Pakkanen 2002, 501–502; Pakkanen 2004a, 268–270; Pakkanen 2011, 148–157; on KDE in general, see Silverman 1986, 7–74; on archaeological analysis and KDE, see Baxter and Beardah 1996; Beardah and Baxter 1999; Baxter 2003, 29–37. Kendall (1974, 245–249) used in his simulations one half of the Gaussian normal distribution, but even though the method fits his case study, it is not a general solution to the production of simulation data sets. Histograms are commonly used as input distributions in computer simulations (see e.g. Law and Kelton 2000, 335–337), but the stepped structure may produce inadvertent quantal qualities in the simulated data sets

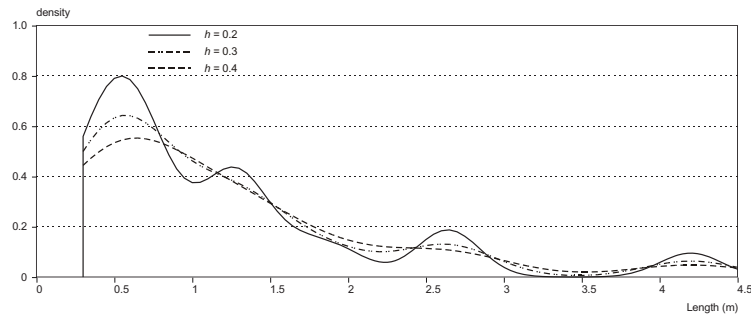


Figure 2.8. Kernel density estimation distributions used to produce simulation data sets. The curves are based on the Erechtheion building block measurements listed in Table 2.1.

The idea behind the KDE is that a small continuous distribution is placed at the position of each observation and these are then added together to create a smooth curve (Figure 2.8). The shape of an individual ‘bump’ can be seen at the right of the figure (solid line). Employing KDE to produce distribution models emphasises the notion that the existing measurements are the most reliable guide to what the general characteristics of the non-quantal data sets should be.<sup>35</sup> In order to avoid producing the quantal properties of the original data, it is necessary to smooth the KDE curve, and this can be done by manipulating the window- or band-width  $h$  which corresponds to the class-width in histograms:<sup>36</sup> when  $h$  is small, the data structure of the original dimensions can be observed more in detail, and when large, the KDE distribution is very smooth (Figure 2.8).

Since the effect of the input distributions on Monte Carlo simulation and cosine quantogram analysis has been questioned by P.R. Freeman,<sup>37</sup> several different KDE distributions with slightly varying band-widths will be used in the following. One thousand simulations are usually regarded sufficient for a statistical test at the 5% level of significance, but I have run three sets of 1,000 simulations for each data model to examine the variation between different Monte Carlo runs.<sup>38</sup> The range for the window-

(Pakkanen 2004a, 267). The computer modules used in the cosine quantogram analysis, Monte Carlo simulations, and producing the KDE distributions have been programmed by the author of this paper on top of Survo MM. Cosine quantogram analysis is now a standard feature of Survo MM (this module has been implemented by Seppo Mustonen).

<sup>35</sup> The parallel with bootstrap-techniques is evident (cf. Manly 1997, 34), though bootstrapping itself cannot be used to produce replica data sets: since bootstrap is based on the possibility of an observation being replicated in the resampled data set, the method produces emphasised quantum peaks which is exactly the opposite than what the properties of a simulation data set should be; Pakkanen 2002, 502; Pakkanen 2004a, 264–266.

<sup>36</sup> The optimal width of  $h$  in the KDEs can be calculated in several different ways. I have used C.C. Beardah’s MATLAB routines to calculate the optimal window-widths of the KDEs; see Baxter and Beardah 1996, 405–408.

<sup>37</sup> Freeman 1976, 23. Freeman’s Bayesian posterior distributions can be shown to be very closely related to Kendall’s cosine quantogram method; see Silverman 1976, 44–45.

<sup>38</sup> On the number of random data sets, see e.g. Manly 1997, 80–84.

Table 2.2. Results of the Monte Carlo simulations ( $n = 1,000$  for each run). The KDE distributions used as simulation data models are based on Table 2.1, Column 3.

1. KDE Distribution	2. $\phi(q)$ , $\alpha = 5\%$	3. $\alpha$ , $\phi(q) = 3.70$
a. $h = 0.2$ , 1st run	3.44	1.4%
b. 2nd run	3.36	1.7%
c. 3rd run	3.38	1.2%
d. $h = 0.3$ , 1st run	3.43	2.1%
e. 2nd run	3.45	1.9%
f. 3rd run	3.41	1.1%
g. $h = 0.4$ , 1st run	3.43	1.4%
h. 2nd run	3.39	1.6%
i. 3rd run	3.38	1.5%
j. Combined results of a–i, $n = 9,000$	3.41	1.5%

widths used in the Erechtheion inscription dimension simulations is 0.2–0.4 (Figure 2.8); the objective values for  $h$  vary between 0.24–0.40.<sup>39</sup>

The results of the Monte Carlo simulations using the different KDE data models are presented in Table 2.2: no differences can be observed between the simulations using the various band-widths to produce the replica data sets; also, discrepancies between the different simulation runs are rather small. All runs have recognised the higher quantum peak of 3.70 at 0.162 m as significant at the 5% level and rejected the second peak at 0.325 m. The results of the different simulations can be combined to obtain more accurate values based on 9,000 runs (line j in Table 2.2): the score for 5% significance level can be determined as 3.41 (the dotted line in Figure 2.7), and the Erechtheion peak height at the half-foot mark of 0.162 m is topped in only 1.5% of the simulations.

Based on comparison of the inscription and the actual block measurements, the 95% bootstrap confidence interval for the Erechtheion foot-standard length can be established as 0.316–0.327 m.<sup>40</sup> It remains to be seen whether cosine quantogram analysis could be used to determine a more precise range than this. Kendall suggests that the precision with which the size of the quantum is known can be calculated as follows: a mathematical model of the data is first used to create a random sample of dimensions  $X$  and quantal properties are introduced to these values by first calculating the nearest integer  $L$  to  $X/q$  and then replacing each  $X$  by

$$X' = Lq + \sigma e, \quad (3)$$

where  $\sigma$  is standard deviation and  $e$  a standardised Gaussian random variable:  $Lq$  can be defined as a quantal target length disturbed by the error  $\sigma e$ . Kendall also showed that the expectation value for the standard deviation  $\sigma$  can be determined as

$$\sigma = \sqrt{q^2 \times \ln(S/\sqrt{2N}) / -2\pi^2}, \quad (4)$$

<sup>39</sup> The band-width  $h$  calculated using Solve-The-Equation method (STE) is 0.236, one-, two- and three-stage Direct-Plug-In (DPI) methods 0.345, 0.305 and 0.269 respectively, Smooth-Cross-Validation (SCV) 0.305, and Normal method 0.396. For the methods, see Baxter and Beardah 1996, 397–408.

<sup>40</sup> See pp. 63–64 below, esp. n. 52.

where  $S$  is the maximum quantum score; the only restriction is that the number of measurements  $N$  should be large.<sup>41</sup> For the Erechtheion block measurements  $q$  is 0.1619 m and  $S$  3.70, so  $\sigma$  can be calculated as 0.0273 m. Since  $N = 21$  and cannot be classified as large, it is necessary to compare the values of sample standard deviation  $s$  for error  $\epsilon$  in formula (1) and  $\sigma$ :  $s$  can be calculated as 0.0282 m, which is almost identical with  $\sigma$ . Therefore, the expectation value  $\sigma$  can be used in the simulations.

The new  $X$  values were produced using a KDE distribution with  $h = 0.3$ . Two hundred new sets of simulated  $X'$ -values were created and analysed using CQG method: the maximum peaks had a range of 0.1604–0.1640 m and standard deviation of 0.0009 m; the 95% confidence interval for the mean can be calculated as 0.3237–0.3244 m. Therefore, based on cosine quantogram analysis of the block dimensions named in *IG I*<sup>3</sup> 474, the best estimate for the Erechtheion foot-unit can be defined with 95% probability as  $324.0 \pm 0.4$  mm. This is approximately 15 times more precise than the initial comparison of block measurements and inscription data would have indicated, so the benefits of employing CQG are quite obvious.

CQG and simulations can also be used to analyse the effect of measurement noise. The data set used in these computer simulations is based on 18 theoretical Skeuotheke inscription dimensions.<sup>42</sup> They are all part of the building plan or individual blocks which could possibly be discovered in a modern excavation of a Greek building, even if it was not well preserved. I have intentionally kept the size of the data set very small: larger data sets are better able to tolerate higher levels of noise. The dimensions and their lengths in feet given in the inscription are listed in Table 2.3. The calculated size in millimetres (Column 3) is based on the above determined Erechtheion average foot length of 0.324 m; actually, any other length within the range of supposed Greek foot units would have served the analysis equally well. One of the measurements, the width of the wall orthostate block, is given in the inscription with the precision of one-sixteenth of a foot,<sup>43</sup> all others in terms of quarter-, half- or a round number of feet.

Figure 2.9 presents a summary of the Skeuotheke simulations. Adding *uniform* noise of  $\pm 10$  mm to the building dimensions does not have any effect on the quantogram analysis: the top two KDE plots in Figure 2.9 show that all 50 simulations picked the quarter-foot mark of c. 81 mm as the quantum (top left) and all had a very high maximum peak score of 4 or more (top right). The second set of simulations demonstrates that even noise at the level of  $\pm 20$  mm has very little effect on the length of detected quantum, though peak scores are significantly lower than in the first set. The vertical line at 3.5 indicates a peak score which most often will be recognized as significant in Monte Carlo simulations, depending of course on the data set: only 22 of 50 simulations produced a peak of 3.5 or higher. As we see in the third set of simulations, a noise-level of  $\pm 30$  mm is enough to collapse the peak at quarter-foot and make the half-foot mark of c. 162 mm the mode of the distribution; 29 simulations peak in the region of 81 and 162 mm, and only a few produce significantly high peaks. Addition of noise at  $\pm 40$  mm and  $\pm 50$  mm gradually diminishes the proportion of correct quanta being detected while there is very little change in the height of maximum peaks produced in the simulations.

<sup>41</sup> Kendall 1974, 253–254, 258–260. More observations increase the reliability of statistical inference, so the larger the sample, the better the results usually are.

<sup>42</sup> *IG II*<sup>2</sup> 1668. For an analysis of the building in the Piraeus and the relationship between the preserved dimensions and the inscription data, see Pakkanen 2002.

<sup>43</sup> The reason for this is that the width is one dactyl or finger-breath wider than the normal wall block width of  $2\frac{1}{2}$  feet.

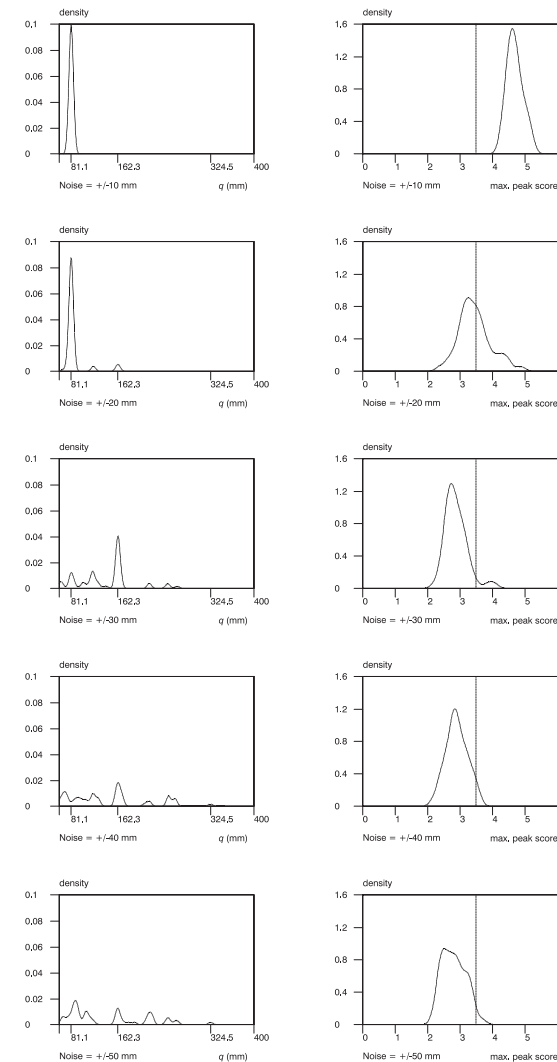


Figure 2.9. KDEs of noise effect on the reliability of CQG analyses based on simulated inscription data ( $n = 50$  for each run). Length of maximum quantum (on the left, band-width  $h = 4$ ) and maximum peak score (on the right,  $h = 0.14$ ).

Table 2.3. Simulated inscription data for Skeuotheke in the Piraeus (1 foot = 0.324 m).

1. Element	2. IG II <sup>2</sup> 1668 (feet)	3. Calculated dimension (m)
Building W	55	17.820
Doorpost L	10	3.240
Doorpost W	2	0.648
Door W	9	2.916
Centre-nave W	20	6.480
Side-nave W	12 <sup>1</sup> / <sub>4</sub>	3.969
Euthyteria block L	4	1.296
Euthyteria block W	3	0.972
Euthyteria block H	1 <sup>1</sup> / <sub>2</sub>	0.486
Corner euthyteria block L	4 <sup>3</sup> / <sub>4</sub>	1.539
Orthostate block L	4	1.296
Orthostate block W	2 <sup>9</sup> / <sub>16</sub>	0.830
Orthostate block H	3	0.972
Wall block W	2 <sup>1</sup> / <sub>2</sub>	0.810
Wall block H	1 <sup>1</sup> / <sub>2</sub>	0.486
Pillar W	2 <sup>3</sup> / <sub>4</sub>	0.891
Pillar stylobate L	4	1.296
Pillar stylobate W	3 <sup>1</sup> / <sub>4</sub>	1.053

Pages 23–109 are missing from this file.



## V. Conclusions

The principal aim of this monograph has been to demonstrate how a more rigorous methodological approach to the question of Greek architectural design could be the starting point towards a scholarly consensus on the topic. It has been my intention to demonstrate why the standard design analysis method has resulted in the current lack of agreement and this can best be done by studying previous case studies in detail. Cosine quantogram analyses and determining the statistical significance levels by subsequent computer simulations is very well-suited to both pinpointing where the shortcomings of the earlier research are and suggesting what the best way forward is. Mathematical algorithms are far better equipped in finding out significant patterns at the core of a complicated set of measurements than intuitive studies. The lack of scholarly agreement is a strong indication that the question of Classical Greek architectural design is a very complex one: the opacity of the design principles behind these masterworks of western architecture makes them an even more fascinating focus of research. I find it intriguing that in order to understand how the Greek architect thought out the design of his buildings we need to resort almost 2,500 years later to quite complicated quantitative analyses. Statistics can also help us to understand what the relationship between the initial plan and the completed structure is and thus enhance our knowledge of the whole process of monumental building. Research in the field and working directly on the building blocks has always filled me with respect of the skill of the ancient mason, but the theoretical analysis is necessary to get closer to the thinking process of the architect. Application of an appropriate rigorous quantitative method should also bring an end to the practice of proposing different hypothetical proportional and modular design schemes based on a *similar* set of dimensions. However, this will not happen until the use of the standard design analysis method comes to an end.

A joint analysis of a group of ten fifth-century Doric temples presented in Chapter III demonstrates that the most constant ratios in the group can be interpreted as an indication of the design rules used by the architect. The building proportions derived from elements at the stylobate level produce systematically higher levels of significance than features further up in the façade. The most probable explanation for this observation is that the Greek temples were designed from bottom up and not top down: they are 'plan-driven' rather than 'façade-driven'. Also, the results of statistical analyses are coherent with Coulton's proposal that the opacity of Doric design is due to successive systems of proportion involving rounding of dimensions at various stages of the design and execution process rather than a strictly modular design system. The exceptionally good fit produced by the proportions expressed in terms of the stylobate block width is consistent with Coulton's suggestion that Vitruvius' term for the module, *embater*, is related to the size of the stylobate block.

When the data sets forming this group are studied building by building, it emerges that only three of the façade measurement sets support a modular hypothesis. These temples could reasonably be described as having a design based on a 'bay module'. A detailed study of the temple of Zeus at Olympia on pages 39–43 reveals that the three highest peaks of the CQG analysis can all be interpreted in terms of the proportional relationships of the stylobate: they match  $\frac{1}{12}$ ,  $\frac{1}{10}$  and  $\frac{1}{7}$  of the stylobate block width and give an indication which fractions can be matched with the façade proportions. All three quanta can also be interpreted not only in terms of the 'Doric' foot of 0.326–0.327 m but also the 'Samian' cubit of 0.521–0.523 m. The analysis of

the temple of the Athenians on Delos (pages 43–46) demonstrates how different data sets result in recognizing various aspects of the building design. The plan dimensions produce a statistically significant peak, but its relationship to the foot-unit is not entirely clear. The façade measurements reflect the proportional relationships between the various parts, but a peak linked with a quarter-foot also emerges. A larger set of plan and elevation dimensions complicates the overall picture, but peaks relating to both proportions and dimensions rounded to quarter and full feet are present. The final data set also confirms that a foot-standard of 0.293–0.294 m was used in the design and execution of the building.

In general, the principal dimensions of the temples discussed in Chapter III are mainly linked with the proportional ratios of the buildings, so metrological studies attempting to establish the lengths of measurement-units used in the Greek world might be more successful by omitting these dimensions from their analyses. Based on the Erechtheion and Tegea studies presented in Chapter IV it is possible to propose that using sets of block measurements will more probably result in the discovery of a statistically valid foot-unit. A quantitative method makes it possible to base the analyses on large data sets, but inclusion of more data can also hide significant patterns, as the temple of the Athenians demonstrates. The detected peaks can, for example, be caused by proportional relationships between the dimensions but also by rounding of the element sizes to the nearest comfortable fraction of a foot.

Chapter IV broadens the range of studied themes beyond fifth-century BC Doric temple design: the four case studies are on the orthogonal town plan of Naxos in Sicily, the Erechtheion on the Athenian Acropolis and two fourth-century temples at Stratos and Tegea. All four sections use a combination of previously published and new data.

The fifth-century grid inside the area of the city walls at Naxos is one of the best preserved examples of regular town planning in the Greek world (pages 56–58). CQG analysis of the new measurements reveals what was the initial design of the orthogonal layout of the city blocks and streets and how this is linked to its execution on the ground. The module is 1.627 m, or in ancient terms five feet of 0.325 m. The execution of the city layout does not precisely follow the original plan in all places: for example, the narrow cross-roads are typically made slightly wider than their modular width and the principal avenues a little narrower. The average discrepancy is as much as 0.19 m (12 per cent of the employed module or nearly two thirds of the foot-unit). Even though the interpretation of the Piraeus case study is not as straight-forward as at Naxos because of the imprecise measurements and small data set, the same module and foot-standard emerge from the analysis of the grid dimensions (pages 53–56). It can be suggested that even though the Sicilian grid employed narrower Archaic proportions for the city blocks than in the ‘Hippodamian’ Piraeus, the two architects employed the same design-unit.

The section on the Erechtheion (pages 60–74) concentrates on determining the length of the foot-standard used in the construction of the temple: using the block inventory inscription (*IG I<sup>3</sup> 474*) as a data selection guide, the length of the foot-unit can with 95% probability be established as  $324.0 \pm 0.4$  mm; the result is further supported by a study of the temple plan dimensions. The case study also shows that discrepancies of up to  $\pm 25$  mm in the execution of the building elements do not prevent CQG analysis from detecting the unit length: the reason is that a sufficient number of blocks were cut reasonably accurately in multiples of half-feet. The key issues in the analyses are the use of an appropriate quantitative method, data selection, and the size of the data set. The standard metrological approach has previously failed to correctly identify the size

of the Erechtheion unit, despite the preserved inventory inscription listing the building block dimensions. The section further emphasises the need of a fresh approach in metrological research: the results reached in earlier studies cannot be taken as the starting point of further analyses and the available data should be thoroughly re-examined using a robust statistical method.

The new fieldwork at Stratos (pages 75–93) strongly suggests that the design of the temple of Zeus was changed in the middle of the building process: the temple was originally planned with one more drum per column shaft. This design change would explain the very conservative proportional height of the temple façade. At the end of the fourth century BC Stratos was under economic strain due to its large-scale urban development programmes: the unfinished temple is a clear indication of the state of the finances of the polis. CQG analysis of the building dimensions reveals a strictly modular design pattern: the length of the module is 0.1053 m. The overall design follows the patterns established for fifth-century monumental architecture in Chapter III: the module is directly related to the stylobate-level dimensions, and the worse fit of the façade dimensions supports the use of a successive proportional design system. The length of the module suggests that the temple construction could be linked with a foot-unit of 0.316 m, but its relationship to the local unit at Stratos cannot be determined on the basis of one structure.

The final case study on the temple of Athena Alea at Tegea presents analyses of the building block dimensions and principal dimensions but it also concentrates on the dimensions of moulding details (pages 94–109). Textual sources on Greek architecture are silent about the measurement-units used in the design and execution of small building elements, but the statistical analysis of the Parthenon mouldings returns a highly significant peak related to the quarter-dactyl of a foot-unit of c. 0.295–0.296 m. The moulding measurements of the temple of Athena Alea do not give a statistically significant result, but a relatively large set of block dimensions indicates a unit of c. 0.099 m. Following the results of the Erechtheion study it very likely arises from the foot-unit employed in the dimensioning of the blocks. If this the case, length of the Athena Alea foot-standard was c. 0.297–0.298 m and, as at Stratos, it suggests that division of feet into thirds instead of quarters was possible in fourth-century architecture. The analysis of the newly established plan dimensions demonstrate that Skopas employed a very unusual design principle for the Late-Classical temple: the stylobate has a simple width to length proportion of 2 to 5 and he derived the differing front and flank column spacings from the stylobate dimensions. The plan is very likely linked with its Archaic predecessor, and even though its anachronistic design features render it difficult to decipher in detail, CQG analysis can reveal the proportional schemes of both the plan and the façade.

All in all, this monograph demonstrates that a shift in design analysis paradigm is in place: there is excellent previous research on Greek architectural design and, especially, the ever expanding series of building monographs provides the essential data to be used in these studies, but the standard methodological approaches are not robust enough to separate between successful and unsuccessful hypotheses. Research employing proper quantitative methods in the analysis of measurement sets can detect meaningful patterns in the data: perhaps one day scholars will be able to agree how the Classical Greek architects designed their buildings.

## Glossary of Architectural Terms

Abacus	The flat element forming the top part of the capital.
Anta	Wall-end, thicker than the rest of the wall. Can also terminate a colonnade: e.g. the porch order columns of a Doric temple are typically <i>in antis</i> , so between two antae.
Architrave	Lintel block carried by columns, also called the epistyle; lowest part of the entablature; see Figure 3.1.
Arris	Sharp edge between two column flutes of a Doric column.
Cella	Central room of a temple.
Column drum	One section of a column shaft; see Figure 3.1.
Embater	Vitruvius' Greek term for the module or basic design-unit of an architectural order (Vitr. 4.3.3).
Entasis	The slightly convex curve of the column taper.
Entablature	Superstructure of a building carried by columns; includes the architrave, frieze and geison; see Figure 3.1.
Epikranitis	The crowning moulding of the cella wall; also used for the top course of blocks of the wall.
Euthyteria	Top course of the foundations, and the very top of the blocks was visible above the ground in antiquity; see Figure 3.1.
Flute	Vertical channel of a column shaft.
Frieze	Central part of an entablature; see Figure 3.1.
Geison	The Greek term for the cornice, the top projecting part of the entablature; see Figure 3.1.
Geison-via	The space between projecting parts of the geison, the mutules.
Krepidoma/krepis	Platform of a temple, usually consisting of three steps; see Figure 3.1.
Metope	Panels of a Doric frieze between the triglyphs; see Figure 3.1.
Opisthodomos	Rear porch of a temple; cf. pronaos.
Orthostate	The lowest course of the walls, upright blocks which are higher than typical wall blocks.
Pronaos	Front porch of a temple enclosed by side walls and with columns in front.
Pteroma	The passage between the exterior colonnade and the cella.
Regula	Rectangular strip under the taenia (the continuous band) of a Doric architrave.
Sima	The gutter of a building, usually of terracotta but in monumental temples can also be carved of marble.
Stylobate	The top step of the krepidoma; see Figure 3.1.
Toichobate	The base of a wall corresponding to the stylobate under the columns.
Triglyph	Tripartite projecting member of a Doric frieze, between the metopes; see Figure 3.1.

## Glossary of Statistical Terms

Band-width ( $h$ )	Factor determining the width of the kernel ('bump') placed at each observation in KDE: when $h$ is small, the data structure of the original dimensions can be observed more in detail, and when large, the KDE curve becomes very smooth. In histograms, the class interval controls similarly the appearance of the plot.
Bayesian posterior distribution	In Bayesian statistics the prior probabilities are modified in light of available data to produce the posterior probabilities. Correspondingly, Bayesian posterior distribution is the distribution (how the set of numerical data are distributed) of a variable based on the available evidence.
Bootstrap techniques	The basic principle behind bootstrap methods is that since the existing sample provides the best knowledge of the studied phenomenon, the sample can be used as a guide to the population distribution. For example, obtaining bootstrap confidence intervals involves taking several random resamples of the sample with replacement in order to approximate the confidence interval range. The name for the technique comes from the saying "to pull oneself up by one's bootstraps".
Confidence interval	A confidence interval for a parameter is an interval calculated on the basis of the sample data so that there is a certain probability, often 95%, that the unknown population mean is within this interval.
Cosine quantogram (CQG) analysis	A statistical method for calculating clustering of data around a particular basic dimension, a quantum, and estimating its statistical significance by computer simulations. See quantum hypothesis below.
Gaussian normal distribution	A common continuous symmetric bell-shaped probability distribution: if the observations are normally distributed, a constant proportion of the cases are between the mean and a certain distance from the mean. The curve is named after the German mathematician C.F. Gauss.
Kernel density estimate (KDE)	An alternative method of displaying a summary of the form of the data to a histogram: it avoids the frequent problems related to the choice of origin in histograms. The principal behind the KDE is that a small continuous distribution is placed at the position of each observation and these are then added together to create a smooth curve; see Figure 2.8.
Monte Carlo simulations	Random computer simulations. In this monograph, Monte Carlo simulations are used to create random samples of data from KDE distributions: the artificial data sets derived from the smooth distributions do not have the quantal properties of the original data

and, therefore, they can be used to test the statistical validity of the highest obtained quantum peaks in CQG, so whether they are statistically significant or not.

#### Quantum hypothesis

The hypothesis assumes that in a data set there is an underlying quantum or basic dimension so that an observation  $X$  can be expressed as the product of an integral multiple  $M$  times the quantum  $q$  plus a small error component  $\varepsilon$  ( $X = Mq + \varepsilon$ ). The clustering around  $q$  can be tested by calculating the CQG score for the data set and, subsequently, Monte Carlo simulations can be used to test the statistical validity of the quantum hypothesis.

#### Window-width

See band-width above.

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