# NLO automated tools for QCD and beyond

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**Abstract** Theoretical predictions for scattering processes with multi-particle final states at next-to-leading order (NLO) in perturbative QCD are essential to fully exploit the physics potential of present and future high-energy colliders. The status of NLO QCD calculations and tools is reviewed.

## 1 Introduction

The study of hard scattering processes at the Large Hadron Collider (LHC) [1] and a future TeV-scale linear collider is our primary means to probe and extend the Standard Model of particle physics. It is driven by the comparison of experimental measurements with theoretical predictions, which depends on our ability to compute collider cross sections in perturbative QCD with adequate accuracy [2, 3]. This can only be achieved by going beyond leading order (LO) in QCD. When using conventional measures, LO scale uncertainties are typically large compared to experimental uncertainties. Moreover, for theoretical reasons a reliable estimation of the scale uncertainty is not feasible at LO. Consequently, an assessment of different scale choices, which is particularly important for many-particle/jet processes, is not possible. Furthermore, the convergence of the perturbative series cannot be assessed at LO. When going beyond LO by including NLO corrections, the situation improves significantly.[2] At NLO, scale uncertainties can be assessed more reliably, and the residual uncertainties are often comparable to experimental uncertainties.[3] NLO calculations thus deliver accurate predictions not only for the overall normalisation, but also for kinematic distributions including peripheral phase space regions. This is in part due to the fact that new subprocesses often become active at NLO, which modify the normalisation and kinematic distributions. Our ability to determine the uncertainty of parton distribution functions (PDF) and to model the structure of jets is also greatly enhanced at NLO.

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[2] For processes with vastly differing scales, the resummation of large logarithms of ratios of scales may also be necessary.

[3] Notable exceptions are the hadroproduction of Higgs and $Wb\bar{b}$ with $\sigma_{\rm NLO}/\sigma_{\rm LO} \approx 2$.

In Section 2, the state-of-the-art methods, implementations and tools for parton-level NLO calculations are briefly reviewed. In Section 3, the status of collider physics applications is described. The review ends with a summary.[4]

## 2 Methods, implementations and tools

The structure and implied modularity of NLO calculations is illustrated in Eqs. (1)–(3):

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{Born}} + \sigma_{corr} \tag{1}$$

$$\sigma_{\mathrm{Born}} = \int d\phi_n \, \frac{1}{2\hat{s}} \quad |\mathcal{A}_{\mathrm{LO}}|^2 \tag{2}$$

$$\begin{aligned}\sigma_{corr} = & \int d\phi_n \, \frac{\alpha_s}{2\hat{s}} \; \Big[ \sum_j \int d\phi_j \mathcal{D}_j + \mathcal{A}_{\mathrm{LO}} \mathcal{A}^*_{\mathrm{NLO,V}} + \mathcal{A}^*_{\mathrm{LO}} \mathcal{A}_{\mathrm{NLO,V}} \Big] \\ & + \int d\phi_{n+1} \, \frac{\alpha_s}{2\hat{s}} \; \Big[ |\mathcal{M}_{\mathrm{NLO,R}}|^2 - \sum_j \mathcal{D}_j \Big] \end{aligned} \tag{3}$$

The new components of the NLO correction $\sigma_{corr}$ are:[5] the virtual corrections (involving one-loop amplitudes), the real corrections (involving tree amplitudes) and the infrared subtraction terms.[6] The resulting procedure for NLO calculations is given in Table 1. The Binoth Les Houches Accord, a standard interface for combining the tree-level and loop-level contributions, has been defined in Ref. [6] and is implemented in many automated tools (see below).

Until circa 2005, the limiting factor of NLO calculations was the computation of the virtual corrections, which typically applied Passarino-Veltman (PV) [7] or PV-inspired [8] tensor integral reduction methods to evaluate the form factors of a Feynman-diagram-based amplitude representation. Several one-loop integral libraries are available as public codes: LoopTools [9, 10], QCDLoop [11], Golem95 [12], OneLOop [13] and PJFry [14]. The PV approach is general, but practical limitations arise due to the factorial growth of the number of Feynman graphs with $N = n + 2$, the strong growth of the number of reduction terms with $N$ and due to numerical instabilities for exceptional kinematic configurations, which are caused by vanishing Gram determinants. It has nevertheless been used successfully to create collections of NLO calculations based on analytic formulae and semi-automated methods, such as MCFM [15, 16], MC@NLO [17] and VBFNLO [18, 19, 20, 21].[7] Since 2004, tremendous improvements have been achieved for the calculation of multi-leg one-loop amplitudes due to the exploitation of on-shell

[4]The important topics of next-to-next-to-leading order (NNLO) calculations and combining parton-level fixed-order calculations and parton-shower event generators are beyond the scope of this review.

[5]The Born amplitude is assumed to be at tree level.

[6]An alternative to the widely used subtraction formalism [4] is the phase space slicing method [5].

[7]The POWHEG BOX [22] library project [23, 24] was inspired by these collections.

| |
|---|
| 1. Real correction: generate and evaluate $2 \to n+1$ tree-level amplitudes |
| 2. Subtract soft and collinear singularities due to single unresolved real radiation to obtain finite result |
| 3. Integrate over $(n+1)$-particle phase space |
| 4. Virtual correction: generate and evaluate UV-renormalised $2 \to n$ one-loop amplitude after extraction of soft and collinear singularities to obtain finite result |
| 5. Confirm cancellation of soft/collinear singularities (absorb initial state collinear singularities into PDF) |
| 6. Integrate over $n$-particle phase space |
| 7. Combine $2 \to n+1$ and $2 \to n$ contributions |
| 8. Convolve with NLO PDF |
| 9. Repeat for all contributing subprocesses |

Table 1: Steps to calculate the NLO QCD corrections for a $2 \to n$ process. $n$ excludes electroweak decays.

recursion relations and generalized-unitarity-cut constructibility as well as the possibility to even reconstruct the full rational terms [25, 26]. On-shell reduction related tools are CutTools [27], Rocket [28] and Samurai [29]. Further innovative, complementary methods are also being developed [30]. A comprehensive review of methods for multi-leg one-loop calculations can be found in Ref. [31].

Three widely-used algorithms for the generation of process-independent infrared subtraction terms are Catani-Seymour dipole subtraction [32], Frixione-Kunszt-Signer (FKS) subtraction [33] and antenna subtraction [34].[8] Several implementations for these standard schemes are available: Sherpa-Dipoles [36], MadDipole [37], HELAC-Dipoles [38], MadFKS [39], TeVJet [40] and AutoDipole [41].

The following programs aim to provide a comprehensive, automated solution for NLO calculations: aMC@NLO [27, 39, 42], BlackHat/Sherpa [26, 36, 43], HELAC-NLO [13, 27, 38, 44], GoSam [45], FeynArts/FormCalc/LoopTools [10, 46] and MadGolem [47].

# 3 Collider physics applications

Discussions at the Les Houches 2005 Physics at TeV Colliders Workshop resulted in a list of processes for which the knowledge of NLO corrections was considered of particular importance

[8] Research on alternative subtraction schemes is also being carried out [35].

for the LHC physics programme [48]. This experimenter's NLO "wish list" has guided theoretical efforts and was subsequently revised and updated in 2007 [49] as well as 2009 [50]. The most recent version is displayed in Table 2.

Due to the groundbreaking advances outlined in Section 2, since 2009 the frontier for collider physics applications of NLO techniques has also advanced considerably. The following $2 \to 4$ processes – most are on the wish list – have now been calculated at NLO QCD:[9] $pp \to W\gamma\gamma$+jet [21], $pp \to W$+3 jets [62, 63, 66, 67], $pp \to Z, \gamma^*$+3 jets [68], $pp \to t\bar{t}b\bar{b}$ [59, 60, 61, 69], $pp \to t\bar{t}jj$ [64, 70], $pp \to b\bar{b}b\bar{b}$ [71], $pp \to W^+W^-b\bar{b}$ [72], $pp \to W^\pm W^\pm jj$ [24, 73], $pp \to W^+W^-jj$ [74] and most recently $pp \to 4$ jets [75]. Leptonic decays of weak bosons can be included trivially. At the same level of complexity, complete off-shell effects for $pp \to t\bar{t}$ with dileptonic decay, i.e. $pp \to e^+\nu_e b\mu^-\bar{\nu}_\mu\bar{b}$, have been calculated at NLO QCD in Ref. [76], which allowed to explicitly confirm the $\mathcal{O}(\alpha_s\Gamma/M)$ effect predicted by Ref. [77]. Advancing the frontier for linear collider physics, the process $e^+e^- \to 5$ jets has recently been calculated at NLO [78], which allowed to extract a competitive value of $\alpha_s(M_Z)$ from 5-jet LEP data. Going beyond 4-particle final states in general requires the computation of 7-point one-loop amplitudes or higher. This is the current complexity frontier. At this level, NLO cross sections in leading-colour approximation have been calculated for $V+4$ jets by the BlackHat/Sherpa collaboration ($pp \to W+4$ jets [79] and $pp \to Z+4$ jets [80]) and for $e^+e^- \to n$ jets up to $n=7$ [81].[10] The $n=7$ case required the computation of a one-loop 8-point function.

# 4 Summary

NLO QCD predictions for multi-particle processes are essential to fully exploit the physics potential of the LHC and a future linear collider. In recent years, tremendous progress has been made in developing the calculational methods and tools that are required to compute NLO corrections for hard scattering processes with 6, 7 or more external particles. At this level a (semi-)manual approach is no longer feasible, and the transition from collections of codes for specific processes to automated code generation for any process up to a maximum complexity has now been achieved. Several such automated tools are available or will become public in the near future. The modularity of NLO calculations allows to interface many tool components on the basis of the Binoth Les Houches Accord.

[9] $pp$ is given as initial state, but $p\bar{p}$ is also implied.

[10] Recently, the full-colour virtual contribution to $pp \to W+4$ jets has been calculated [82].

| Process ($V \in \{Z, W, \gamma\}$) | Comments |
|---|---|
| Calculations completed since Les Houches 2005 | |
| 1. $pp \to VV$+jet | $WW$+jet completed by Dittmaier/Kallweit/Uwer [51, 52]; Campbell/Ellis/Zanderighi [53]. $ZZ$+jet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [54] |
| 2. $pp \to$ Higgs+2jets | NLO QCD to the $gg$ channel completed by Campbell/Ellis/Zanderighi [16]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [55, 56] |
| 3. $pp \to V\,V\,V$ | $ZZZ$ completed by Lazopoulos/Melnikov/Petriello [57] and $WWZ$ by Hankele/Zeppenfeld [19] (see also Binoth/Ossola/Papadopoulos/Pittau [58]) |
| 4. $pp \to t\bar{t}\,b\bar{b}$ | relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini [59, 60] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [61] |
| 5. $pp \to V$+3jets | calculated by the Blackhat/Sherpa [62] and Rocket [63] collaborations |
| Calculations remaining from Les Houches 2005 | |
| 6. $pp \to t\bar{t}$+2jets | relevant for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek [64] |
| 7. $pp \to VV\,b\bar{b}$, | relevant for VBF $\to H \to VV$, $t\bar{t}H$ |
| 8. $pp \to VV$+2jets | relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [20] |
| NLO calculations added to list in 2007 | |
| 9. $pp \to b\bar{b}b\bar{b}$ | $q\bar{q}$ channel calculated by Golem collaboration [65] |
| NLO calculations added to list in 2009 | |
| 10. $pp \to V$+4jets | top pair production, various new physics signatures |
| 11. $pp \to Wb\bar{b}j$ | top, new physics signatures |
| 12. $pp \to t\bar{t}t\bar{t}$ | various new physics signatures |
| Calculations beyond NLO added in 2007 | |
| 13. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$ | backgrounds to Higgs |
| 14. NNLO $pp \to t\bar{t}$ | normalisation of a benchmark process |
| 15. NNLO to VBF and $Z/\gamma$+jet | Higgs couplings and SM benchmark |
| Calculations including electroweak effects | |
| 16. NNLO QCD+NLO EW for $W/Z$ | precision calculation of a SM benchmark |

Table 2: The experimenter's wish list for LHC processes in early 2010 (from [50]).

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